Giant phase-conjugate reflection with a normal mirror in front of an optical phase-conjugator

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We theoretically study reflection of light by a phase-conjugating mirror preceded by a partially reflecting normal mirror. The presence of a suitably chosen normal mirror in front of the phase conjugator is found to greatly enhance the total phase-conjugate reflected power, even up to an order of magnitude. Required conditions are that the phase-conjugating mirror itself amplifies upon reflection and that constructive interference of light in the region between the mirrors takes place. We show that the phase-conjugate reflected power then exhibits a maximum as a function of the transmittance of the normal mirror.

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In recent years, the interesting role that phase conjugation can play in mesoscopic physics has received considerable attention. Phase conjugation is the general term for a process in which both the direction of propagation and the overall phase factor of a wave function are reversed [1]. A famous example is Andreev reflection [2], the electron-to-hole reflection at an interface between a normal metal and a superconductor in a mesoscopic system. Mesoscopic means, by definition, that the dimensions of the system are small enough for phase-coherence to be preserved in the entire configuration, in this case the normal metal–superconductor junction, but still much larger than the Fermi wavelength of the normal metal. In this regime it is interesting to study the influence of Andreev reflection on transport properties such as the conductance of the sample. The basis of a theoretical approach to this kind of transport problems was in fact laid as early as 1957 by Landauer, long before the advent of mesoscopic physics. He related the electrical conductance of a normal metal to its quantum mechanical transmission matrix, the well-known Landauer formula [3,4]. This reflection/transmission matrix approach turned out to be very useful in mesoscopic transport problems, also in combination with phase conjugation. For example, Beenakker [5] derived a "Landauer-type" formula for the conductance of a normal metal-superconductor junction, which allows for studying both clean and disordered normal metals in the presence of Andreev reflection.

This paper is concerned with the optical counterpart of Andreev reflection, namely probe-to-conjugate reflection of light at a phase-conjugating mirror (PCM) [1]. The dimensions of a PCM are much larger than those of a mesoscopic superconducting system and allow for a classical treatment of the transport properties of light. Despite this different scale, a PCM system is mesoscopic in the sense that it is larger than the wavelength, but smaller than the coherence length of the light reflecting at it. Electronic and optical phase conjugation display an interesting analogy [6,7] and also optical phase conjugation can conveniently be described using a reflection/transmission formalism. Recently this was used to study reflection of light at a phase-conjugating mirror behind a disordered optical medium in a waveguide [8].

In view of the time-reversal properties of a phase-conjugating mirror, whereby accumulated phase-shifts are cancelled when the phase-conjugated wave travels back along the path of the original incoming wave, it is intruiging to combine it with a normal reflector. By normal reflector we mean a scattering region in which only specular reflections take place, eg. the disordered medium in [8]. Here we consider the simplest possible normal reflector, a partially transmitting normal mirror. We analyze the phase-conjugate reflected power for plane wave illumination of a configuration consisting of a phase-conjugating mirror preceded by a partially reflecting normal mirror. A lot of work has been done on this kind of resonator structures involving combinations of normal mirrors and phase-conjugating mirrors [9]. Here we describe an interesting effect in a normal mirror–phase-conjugating mirror arrangement, which to our knowledge has been unnoticed so far. When the PCM is operating such that it amplifies the incoming light upon phase-conjugate reflection and multiply reflected waves in the region between the mirrors interfere constructively, we find a dramatic enhancement of the phase-conjugate reflected intensity compared with that at the same PCM alone. The phase-conjugate reflectance, defined as the reflected power divided by the incident power, is maximal for a suitably chosen value of the transmittance of the normal mirror and reaches ~ 10 times its value at the PCM alone.

The phase-conjugating mirror consists of a cell filled with an optical medium with a large third-order susceptibility $\chi^{(3)}$. The medium is pumped by two intense counterpropagating laser beams of frequency ω_0. When a weak probe beam of frequency $\omega_0 + \delta$ is incident on the material, a fourth beam will be generated due to the nonlinear polarization of the medium (four-wave mixing). This so-called conjugate wave propagates with frequency $\omega_0 - \delta$ in the opposite direction as the probe beam [1]. In the medium, the coupling between probe and conjugate waves is described by the equations [6]

$$\begin{pmatrix} -\frac{c^2}{2\omega_0}\frac{\partial^2}{\partial x^2} - \frac{\omega_0}{2} & -\gamma \\ \gamma^* & \frac{c^2}{2\omega_0}\frac{\partial^2}{\partial x^2} + \frac{\omega_0}{2} \end{pmatrix} \begin{pmatrix} \mathcal{E}_p(x) \\ \mathcal{E}_c^*(x) \end{pmatrix} = \delta \begin{pmatrix} \mathcal{E}_p(x) \\ \mathcal{E}_c^*(x) \end{pmatrix}. \tag{1}$$

Here $\mathcal{E}_{p(c)}(x)$ denotes the complex amplitude of the probe (conjugate) field and $\gamma \equiv \gamma_0 e^{i\phi}$ is the pumping induced coupling strength between the two fields. Outside the medium $\gamma = 0$ and (1) reduces to two uncoupled equations for the probe and conjugate waves.

The one-dimensional system we consider is schematically depicted in Fig. 1. A phase-conjugating mirror is preceded at distance L by, in general, an elastic scattering region (SR) which does not couple probe and conjugate waves.

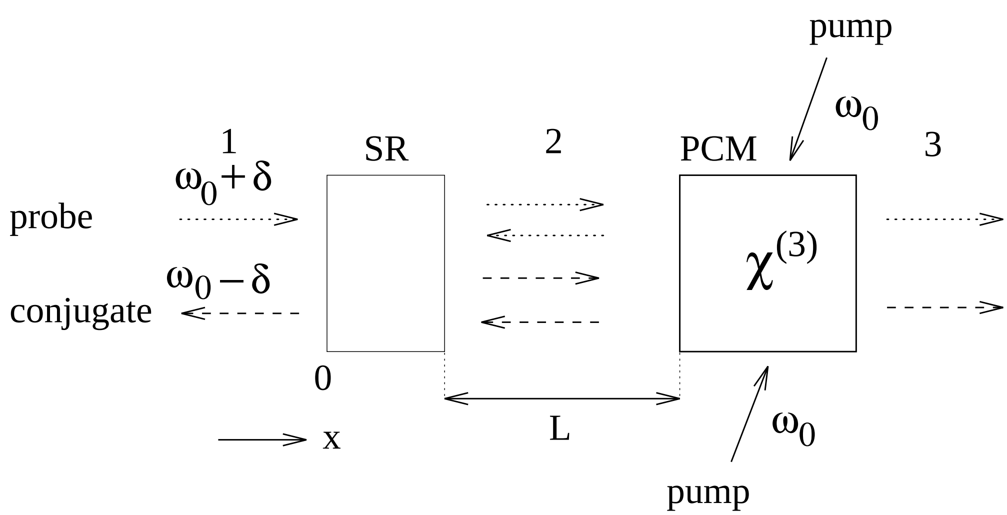

FIG. 1. Reflection and transmission in one dimension at a scattering region (SR) followed by a phase-conjugating mirror (PCM). Dotted (dashed) arrows denote probe (conjugate) waves.

We are interested in the phase-conjugate reflected intensity at $x = 0$ if a probe beam is incident on the region from the left. This can be straightforwardly calculated using a scattering matrix formalism. Let $\mathcal{E}^{+}_{p(c),1(2)}$ denote the electric field amplitude of a probe (conjugate) wave travelling to the right (left) in region 1 (2). We then have

$$\begin{pmatrix} \mathcal{E}^{+}_{p,3}(x) \\ \mathcal{E}^{-}_{p,3}(x) \\ \mathcal{E}^{-}_{c,3}(x) \\ \mathcal{E}^{+}_{c,3}(x) \end{pmatrix} = S_{pcm}\, U(x)\, S(x) \begin{pmatrix} \mathcal{E}^{+}_{p,1}(x) \\ \mathcal{E}^{-}_{p,1}(x) \\ \mathcal{E}^{-}_{c,1}(x) \\ \mathcal{E}^{+}_{c,1}(x) \end{pmatrix}, \tag{2}$$

with S_{pcm} the scattering matrix of the PCM, S that of the scattering region in front and U the transfer matrix in region 2. From the solution of the matrix equation (2) for known S and S_{pcm} the phase-conjugate reflected intensity $R_c \equiv |\mathcal{E}^{+}_{c,1}(0)|^2$ at $x = 0$ is obtained. In general R_c can be written as

$$R_c = \frac{T_p\, T_c\, R_{pcm}}{1 + (1 - T_p)(1 - T_c)R^2_{pcm} - 2R_{pcm}\left[\mathrm{Re}(r_{p,2} \cdot r_{c,2}) \cos\phi - \mathrm{Im}(r_{p,2} \cdot r_{c,2}) \sin\phi\right]}. \tag{3}$$

Here $T_{p(c)}$ denotes the transmittance of probe (conjugate) waves through the scattering region and $\mathrm{Re}\,(r_{p,2})$ is the real part of the amplitude of probe-to-probe reflection at SR in region 2, etc. ϕ is the phase accumulated during multiple reflections in region 2 [10],

$$\phi = 4\frac{\delta}{c}L + 2\arctan\left(\frac{\delta}{\sqrt{\delta^2 + \gamma_0^2}} \tan(\beta L_{pcm})\right) \tag{4}$$

and R_{pcm} is the phase-conjugate reflected power at the PCM alone [1],

$$R_{pcm} = \frac{\sin^2(\beta L_{pcm})}{\cos^2(\beta L_{pcm}) + (\frac{\delta}{\gamma_0})^2}, \tag{5}$$

with $\beta = \sqrt{\delta^2 + \gamma_0^2}/c$ and δ the detuning frequency between probe and pump waves, $\delta \ll \omega_0$. If the scattering region consists of a single partially transmitting normal mirror, (3) reduces to

$$\begin{aligned} R_c &= \frac{T^2 R_{pcm}}{1 + (1-T)^2 R_{pcm}^2 - 2(1-T) R_{pcm} \cos\phi} \\ &= \frac{T^2 R_{pcm}}{\left[1 - (1-T) R_{pcm}\right]^2 + 2(1-T) R_{pcm} (1 - \cos\phi)}, \end{aligned} \tag{6}$$

with $T \in [0,1]$ the transmittance of the normal mirror at frequency ω_0 [11]. Fig. 2 shows R_c as a function of T for various values of the intermirror distance L.

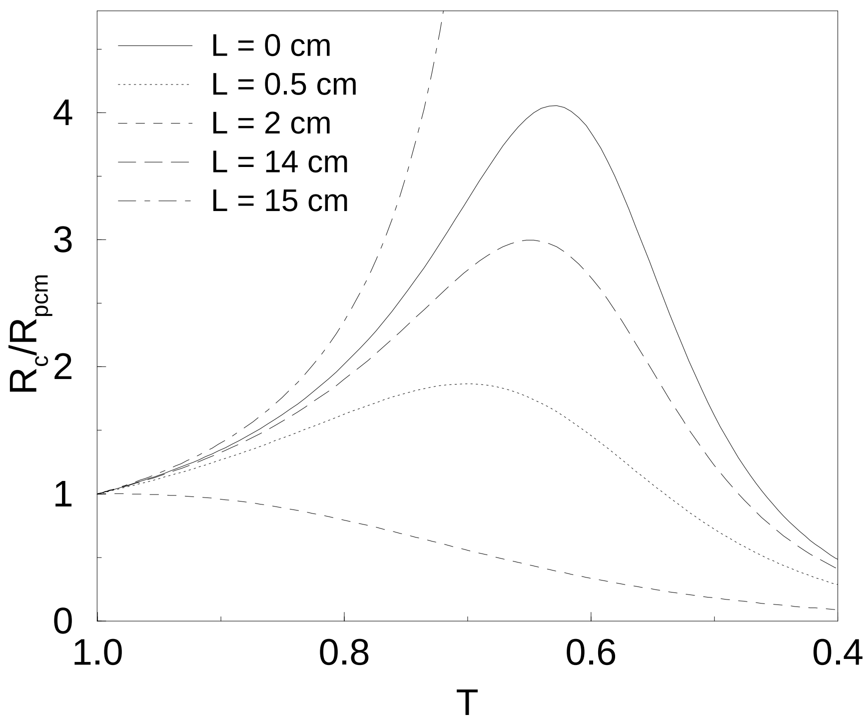

FIG. 2. Phase-conjugate reflectance at a normal mirror with transmittance T followed by a phase-conjugating mirror for $L_{pcm} = 1\,cm$, $\delta = 0.1\,[c/L_{pcm}]$, $\gamma_0 = 1\,[c/L_{pcm}]$. The phase-conjugate reflectance of the PCM alone is $R_{pcm} = 2.4$

We see that the three middle curves in Fig. 2 display a maximum as a function of T. More precisely, the maximum in the phase-conjugate reflected intensity occurs if $R_{pcm} \cos\phi \geq 1$ and for normal mirror transmittance

$$T_{max} = \frac{1 - 2R_{pcm}\cos\phi + R_{pcm}^2}{R_{pcm}(R_{pcm} - \cos\phi)}. \tag{7}$$

R_c then reaches a value which amounts to several times the phase-conjugate reflected intensity at a PCM alone.

Placing a normal mirror in front of a phase-conjugating mirror thus greatly enhances the phase-conjugate reflected intensity, as long as the PCM acts as an amplifier ($R_{pcm} > 1$) and the phase ϕ is such that $\cos\phi \sim 1$. The maximum occurs because of two competing effects: on the one hand, the normal mirror causes direct back-reflection of part of the incoming probe wave, so that less light reaches the PCM. On the other hand it opens the possibility for multiple reflections in the region between the two mirrors. For large normal mirror transmittance the latter effect is dominant, because of the gain in intensity for each reflection at the PCM and the constructive interference of light in the resonator ($\phi \sim 2\pi n$). As T becomes smaller, however, the increasing loss of light through backscattering without phase-conjugation causes R_c to drop again. The amplifying property of the PCM is essential for obtaining the maximum. This can easily be seen from (6) which shows that $R_c < R_{pcm}$ for $R_{pcm} < 1$ [12]. The constructive interference of multiply reflected waves is also essential for obtaining the maximum. Fig. 2 shows that for $L = 2\,cm$ (or, equivalently, $\phi \approx \pi/3$), R_c always decreases with T, even though the PCM still acts as an amplifier. The advantage of additional multiple reflections, as the transmittance of the normal mirror becomes less, then cannot overcome the disadvantage of less primary light arriving at the PCM. Note that a finite distance L is not necessary for obtaining the maximum. The curve for $L = 0$ shows that a suitably chosen coating on the PCM also gives rise to enhancement of R_c. In this case the effect is very sensitive to the detuning frequency δ which, as seen from (6) with $L = 0$, has to satisfy the condition $\delta \ll \gamma_0$ in order for ϕ to be close to zero.

From (6) we see that R_c becomes infinite if $\cos\phi = 1$ and $T = 1 - 1/R_{pcm}$. This happens in the upper curve of Fig. 2, for $L = 15\,cm$. It is interesting to compare this with the work of Paasschens *et al.* who obtained qualitatively the same divergent behavior in a more complex system [8]. They studied reflection of light at a two-dimensional random medium consisting of dielectric rods backed by a phase-conjugating mirror. For diffusive illumination of the disordered medium and for $\delta \gg \tau_{dwell}^{-1}$, with τ_{dwell} the dwell time of a photon in the medium, interference effects are shown to be negligible. Disregarding angular correlations between multiply reflected waves in the region between the random medium and the PCM then leads to a phase-conjugate reflected power of

$$R_c = \frac{T^2 R_{pcm}}{1 - (1-T)^2 R_{pcm}^2}. \tag{8}$$

The same expression is obtained when averaging (6) over the phase ϕ [13]. (8) again shows the divergence of R_c for $T = 1 - 1/R_{pcm}$. As was explained in [8], this is due to some light becoming trapped in the intermirror region as T decreases. Repetitive reflections at the PCM, provided $R_{pcm} > 1$, then lead to the enormous increase in the phase-conjugate reflectance. Since R_c is limited by the intensity of the pump beams, one now needs to take into account the effects of pump depletion and the analysis breaks down close to and beyond $T = 1 - 1/R_{pcm}$.

The same phenomenon occurs in our one-dimensional system for certain values of L [14]. This is illustrated in Fig. 3, which shows the phase-conjugate reflected intensity as a function of the intermirror distance L for a normal mirror with transmittance of 60 %. The horizontal line in the figure indicates the reflectance of the PCM alone, R_{pcm}.

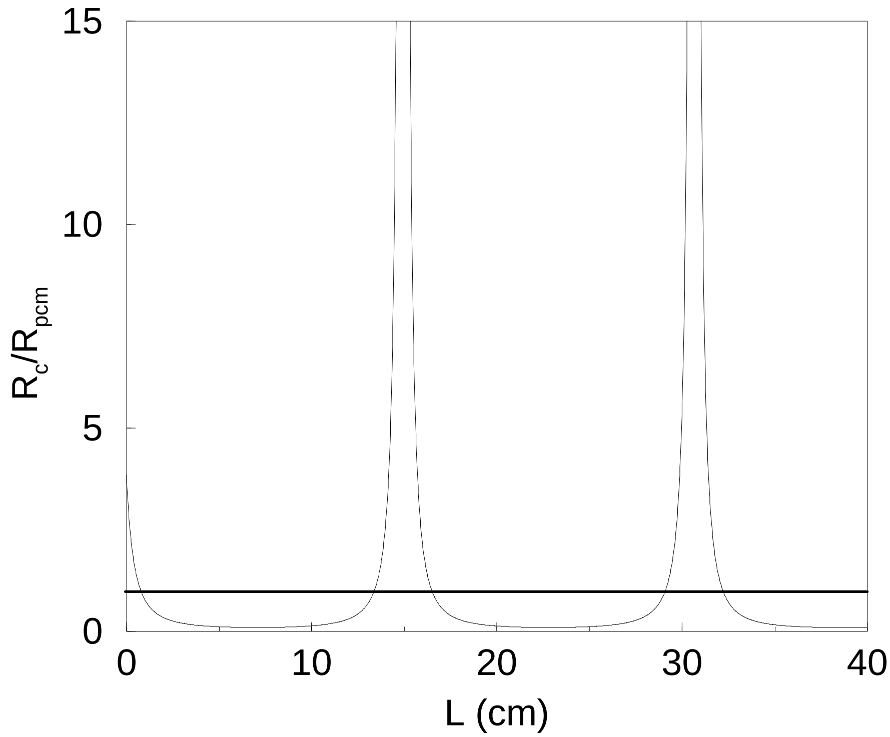

FIG. 3. Phase-conjugate reflectance at a normal mirror with transmittance $T = 0.6$ followed by a phase-conjugating mirror. Parameters used are $L_{pcm} = 1\,cm$, $\delta = 0.1\,[c/L_{pcm}]$ and $\gamma_0 = 1\,[c/L_{pcm}]$. The horizontal line marks the phase-conjugate reflectance at the PCM alone, $R_{pcm} = 2.4$.

It is clear that the presence of the normal mirror at a suitably chosen distance L, such that ϕ is close to a multiple of 2π, but sufficiently far away from it to avoid the effects of pump depletion, gives a giant enhancement of R_c with respect to R_{pcm}.

In conclusion, we have theoretically analyzed the phase-conjugate reflected power at a normal mirror followed by a phase-conjugating mirror. For plane wave illumination of this mirror arrangement, and taking advantage of multiple reflection effects in the resonator region between the mirrors, a giant enhancement of the phase-conjugate reflectance is predicted with respect to that of the PCM alone. Necessary conditions are that reflection at the phase-conjugating mirror is accompagnied by amplification, and that constructive interference of waves occurs in front of the PCM. If either condition is not satisfied, the presence of the normal mirror only decreases the phase-conjugate reflectance. This also happens in the analogous electronic configuration, a normal metal-superconductor interface preceded by a normal scattering region: since Andreev reflection occurs with at most unity amplitude, the hole reflectance decreases with increasing normal scattering [15].

In previous work on resonators involving a normal mirror and a phase-conjugating mirror [9] various ways of output power enhancement of the resonator were found. For example, Pepper *et al.* [16] reported amplified phase-conjugate reflection using degenerate four-wave mixing ($\delta = 0$) and a fully reflecting normal mirror in combination with a PCM. Under suitably chosen operating conditions of the PCM they observed a finite conjugate output signal in the absence of any input probe signal, the output being directly generated by

the pump beams [17]. Feinberg *et al.* [18] obtained similar self-oscillatory behavior using a perfectly reflecting normal mirror behind a PCM.

Our configuration differs from these resonators, since it gives the phase-conjugate response to an input probe signal passed through a partially transmitting normal mirror. Enhancement of this response occurs both for frequencies $\delta = 0$ (degenerate four-wave mixing) and $\delta \neq 0$ (non-degenerate four-wave mixing), and in a broad range of intermirror distances L. This allows for frequency discrimination between probe and conjugate output waves and regularization of the enhancement by choosing an appropriate value of L for fixed δ, or vice versa.

Finally, as an example, consider a realistic phase-conjugating mirror of length $L_{pcm} = 1\,cm$, with a coupling strength $\gamma_0 = 10^{10}s^{-1}$ and illuminated by monochromatic light of frequency $\delta \sim 10^9 s^{-1}$ detuned from the pump frequency. The phase-conjugate reflectance $R_{pcm} \sim 2$, which is well within experimental reach [19]. We predict that placing a $\approx 60\%$ transmitting normal mirror at a distance of $L \approx 14\,cm$ in front of this PCM increases the phase-conjugate reflected power by an order of magnitude and hope that this will present a challenge to experimentalists.

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[10] (4) is the condition for the formation of a bound state in a normal mirror–phase-conjugating mirror resonator [7]. Due to the coupling of probe and conjugate waves by the PCM, a complete roundtrip requires four passages through the resonator region (first term on right-hand side of (4)) and two reflections at the PCM (second term).

[11] We neglect the difference in normal-mirror transmission of probe and conjugate waves here, since $\delta \ll \omega_0$.

[12] If reflection at the PCM is not accompagnied by amplification, enhancement of the phase-conjugate reflectance can be achieved by placing a normal mirror behind the phase-conjugating mirror and illuminating the PCM with probe light in front. The total phase-conjugate reflectance at the PCM then consists of a part resulting from direct phase-conjugate reflection at the PCM plus an extra part as a result of multiple reflections in the region between phase-conjugating mirror and normal mirror.

[13] In performing this average we in fact consider a somewhat hypothetical "two-dimensional" system consisting of a sum of N one-dimensional channels (eg. fibres), each containing a normal mirror, in front of the PCM. If the positions of the normal mirrors in each of these fibres are random, the accumulated phases $\phi_n, n = 1 \ldots N$ will vary uniformly over integral multiples of 2π for $N \gg 1$, and one can average over them.

[14] Notice that the behavior of the phase-conjugate reflectance before the enormous increase at $T = 1 - 1/R_{pcm}$ is different in the one-dimensional and two-dimensional situation. In one dimension R_c immediately increases as T becomes less than 1 (upper curve in Fig. 2). In two dimensions it first decreases and displays a minimum at $T = 1 - 1/R_{pcm}^2$ [8] as a result of the diffusive illumination and subsequent angular, or phase-shift average.

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